Misspecification and Weak Identification in Asset-Pricing Models

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1 Preliminaries
2 Setup
3 Severity of the problem
4 What has been done
5 Conclusion
All economic models should be viewed as approximations to the true data generating process (Watson, 1993; White, 1982, 1994; Canova, 1994; among others)
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“Misspecification of these models is therefore endemic and inevitable. Omission of relevant variables, inclusion of irrelevant variables, incorrect functional forms, incompleteness of systems of relations, and incorrect distributional assumptions are both common and present simultaneously” (Maasoumi, 1990).

As a result, standard error (confidence interval) construction and model comparison should allow for possible model misspecification. Still, it is common practice to use GMM standard errors for correctly specified models even when the model is rejected by the data.
Asset returns are very noisy and only weakly correlated with the much less volatile macroeconomic (non-traded) factors. The average (absolute) correlations of 25 FF portfolio monthly returns with $vw$ (0.82), $c_{nd}$ (0.14), $c_d$ (0.05), $lab$ (0.04), $c \cdot cay$ (0.01)
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- The pseudo-true value of the parameter associated with the spurious factor is not identifiable
- Standard tests of significance substantially overreject under the null and one may erroneously conclude that the spurious factor is priced
Define spurious factors as factors that are independent of the returns on the test assets and the remaining factors included in the model.

It is tempting to conjecture that the presence of spurious factors is not a major concern in empirical work because:
- It can be detected by a test of significance.
- It may not affect the analysis of the remaining risk premia parameters.

It turns out that this type of argument is ill-founded because:
- A standard pre-test of statistical significance will erroneously conclude, with high probability, that the spurious factor is relevant and should be included in the model.
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Stochastic discount factor (SDF) approach to asset pricing

\[ E[R_t m_t] = 1_N, \]

where \( m_t \) is an admissible SDF and \( R_t \) are gross returns on \( N \) test assets.

The continuously-updated (CU-)GMM estimator of \( \lambda \) is defined as

\[ \hat{\lambda} = \arg\min_{\lambda} \bar{e}(\lambda)' \hat{W}(\lambda) - 1 \bar{e}(\lambda), \]

\[ \bar{e}(\lambda) = \frac{1}{T} \sum_{t=1}^{T} e(t)(\lambda), \]
\[ \hat{W}(\lambda) = \frac{1}{T} \sum_{t=1}^{T} (e(t)(\lambda) - \bar{e}(\lambda))(e(t)(\lambda) - \bar{e}(\lambda))'. \]

The over-identifying restriction test of the asset-pricing model is

\[ J(\lambda) = T \min_{\lambda} \bar{e}(\lambda)' \hat{W}(\lambda) - 1 \bar{e}(\lambda), \]
\[ J(\hat{\lambda}) \rightarrow_d \chi^2_{N-K} \]
when the asset-pricing model holds.
SDF approach to asset pricing: CU-GMM framework

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  and \( J(\hat{\lambda}) \to^d \chi^2_{N-K} \) when the asset-pricing model holds.
It is often desirable to estimate and evaluate the asset-pricing model in the beta-pricing setup

\[ R_t - \mu = \beta (f_t - \mu_f) + \epsilon_t, \quad t = 1, \ldots, T, \]

where \( \mu = E[R_t], \mu_f = E[f_t] \) and \( \beta \) is an \( N \times (K - 1) \) matrix.
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The beta-pricing model suggests that \( \mu = 1_N \gamma_0^* + \beta \gamma_1^* \), where \( \gamma_1 \) has a direct interpretation of a vector of risk premium parameters.
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Assuming that \( \epsilon_t \sim iid \mathcal{N}(0_N, \Sigma) \) conditional on \( f_t \), the MLE of \( \gamma \) is

\[
\hat{\gamma} = \arg\min_{\gamma} \frac{(\hat{\mu} - 1^N\gamma_0 - \hat{\beta}\gamma_1)'\hat{\Sigma}^{-1}(\hat{\mu} - 1^N\gamma_0 - \hat{\beta}\gamma_1)}{1 + \gamma_1'\hat{V}_f^{-1}\gamma_1},
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where \( \hat{\mu}, \hat{\beta}, \hat{V}_f, \hat{\Sigma} \) are sample estimators of \( \mu, \beta, V_f = \text{Var}[f_t], \Sigma \).
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where \( \hat{\mu}, \hat{\beta}, \hat{V}_f, \hat{\Sigma} \) are sample estimators of \( \mu, \beta, \text{Var}[f_t], \Sigma \).

The test of correct model specification is given by

\[
S = TQ(\hat{\gamma}) \overset{d}{\rightarrow} \chi^2_{N-K}
\]

and the fitted returns are constructed as \( \hat{\mu} = 1_N \hat{\gamma}_0 + \hat{\beta} \hat{\gamma}_1 \).
Main statistics of interest

1. $t$-statistics of statistical significance which provide evidence on whether
   - the factor is priced (SDF framework, $H_0: \lambda_i = 0$)
   - the risk premium is nonzero (beta-pricing model, $H_0: \gamma_i = 0$)

2. Test for correct model specification
   - $J$ test for overidentifying restrictions (SDF: $H_0: D\lambda - 1_N = 0_N$)
   - $S$ (Shanken’s) test (beta-pricing model, $H_0: \mu - 1_N\gamma_0 - \beta'\gamma_1 = 0_N$)

3. Goodness-of-fit (pseudo-$R^2$) measure: squared correlation between actual and fitted ($\hat{\mu} = 1_N\hat{\gamma}_0 + \hat{\beta}'\hat{\gamma}_1$) returns
   - the fitted returns in the SDF approach are constructed using the mapping between the SDF and beta-pricing model parameters

\[ \gamma_0 = \frac{1}{\lambda_0 + \mu'_f\lambda_1}, \quad \gamma_1 = \frac{-V_f\lambda_1}{\lambda_0 + \mu'_f\lambda_1} \]
Empirical evidence: CAPM

Empirical results from estimating CAPM (market factor) (test assets: 25 Fama-French portfolio returns, 1959:02 – 2012:12)

<table>
<thead>
<tr>
<th>t-stat</th>
<th>spec. test</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>3.24</td>
<td>67.66</td>
</tr>
<tr>
<td>(0.0012)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>CU-GMM</td>
<td>4.29</td>
<td>64.58</td>
</tr>
<tr>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The specification test is the $S$ test (ML) or $J$ test (CU-GMM) of the null of correct model specification. The $R^2$ is computed as the squared correlation of the actual and fitted returns. $p$-values in parentheses
Realized vs. fitted returns: CAPM

ML

45° line
fitted returns

CU-GMM

45° line
fitted returns

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<table>
<thead>
<tr>
<th></th>
<th>market factor</th>
<th>“mysterious” factor</th>
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<tr>
<td></td>
<td>t-stat</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ML</td>
<td>1.03 (0.3030)</td>
<td>4.61 (0.0000)</td>
<td>19.94 (0.5871)</td>
<td>0.9972</td>
</tr>
<tr>
<td>CU-GMM</td>
<td>1.36 (0.1738)</td>
<td>4.42 (0.0000)</td>
<td>18.07 (0.7019)</td>
<td>0.9542</td>
</tr>
</tbody>
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Empirical evidence: CAPM + “mysterious” risk factor

Robotti (2015)
Empirical results from estimating a model with a “mysterious” risk factor
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<td>4.67</td>
<td>21.36</td>
<td>0.9998</td>
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<td></td>
<td>(0.0000)</td>
<td>(0.5592)</td>
<td></td>
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<td>19.95</td>
<td>0.9941</td>
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<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.6447)</td>
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The results from this empirical exercise are striking. The model exhibits almost perfect fit ($R^2 \approx 1$). Based on the $S$ and $J$ tests, the model appears to be correctly specified. The risk factor (premium) is highly significant and is deemed to be priced (nonzero). What is this mysterious factor? The risk factor is generated as a standard normal random variable which is independent of returns! The results are completely spurious; the true model is misspecified, the factor is unpriced, and $R^2 = 0$. In summary, an arbitrarily bad model with one or more factors that are independent of asset returns (spurious factors) is concluded to be a correctly specified model with a spectacular fit and priced risk factors. Even worse, the priced factors that are highly correlated with returns are driven out (become insignificant) when a spurious factor is included in the model.
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Further empirical evidence: ML

- These results also arise in popular empirical models
  - C-LAB: market factor, growth rate of per capita labor income ($labor$), and the lagged default premium ($prem$)
  - CC-CAY: real per capita consumption growth ($cg$), the lagged consumption-aggregate wealth ratio ($cay$), and an interaction term between these two factors ($cg \cdot cay$)

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<td>$S$ ($p$-value)</td>
<td>23.10 (0.3388)</td>
<td>11.58 (0.9503)</td>
</tr>
<tr>
<td>$t_{market}$</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>$t_{labor}$</td>
<td>2.81</td>
<td></td>
</tr>
<tr>
<td>$t_{prem}$</td>
<td>4.21</td>
<td></td>
</tr>
<tr>
<td>$t_{cg}$</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>$t_{cay}$</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>$t_{cg \cdot cay}$</td>
<td>3.45</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9994</td>
<td>0.9997</td>
</tr>
</tbody>
</table>
Robotti (2015)
In all of these examples (model with “mysterious” factor, C-LAB, CC-CAY), there is identification failure.

In the SDF framework, the matrix

\[
\frac{\partial \bar{e}(\lambda)}{\partial \lambda'} = E[R_t \tilde{f}_t'] = D
\]

is of reduced rank because one or more factors are uncorrelated (or weakly correlated in finite samples) with returns.

The rank test (Cragg-Donald) cannot reject the null that the second moment matrix of the returns and the factors is of reduced rank.

For example,

<table>
<thead>
<tr>
<th></th>
<th>CAPM</th>
<th>FF3</th>
<th>C-LAB</th>
<th>CC-CAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank test (p-value)</td>
<td>133.43 (0.0000)</td>
<td>86.18 (0.0000)</td>
<td>20.82 (0.5320)</td>
<td>10.44 (0.9818)</td>
</tr>
</tbody>
</table>
Some findings

1. The power of the specification tests is equal to their size as $T \to \infty$.
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   - i.e., the model is deemed to be correctly specified with high probability even when the degree of misspecification is arbitrarily large.
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3. Risk factors that are spurious are deemed to be priced with probability approaching one
   - The $t$-tests associated with the spurious factor are asymptotically distributed as $\chi^2_{N-K+1}$ instead of $\chi^2_1$

4. Risk factors that are useful and priced are likely to be deemed unpriced
   - The $t$-tests of statistical significance for the useful factors are inconsistent (i.e., converge to bounded random variables) and exhibit power that is close to the size of the test
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Limiting densities of the $t$-statistics: CU-GMM

Simulation evidence: design

- Three linear models: (1) with a useful factor only, (2) with a spurious factor only, and (3) with both factors.
Simulation evidence: design

- Three linear models: (1) with a useful factor only, (2) with a spurious factor only, and (3) with both factors
- The models can be correctly specified or misspecified
Simulation evidence: design

- Three linear models: (1) with a useful factor only, (2) with a spurious factor only, and (3) with both factors
- The models can be correctly specified or misspecified
- The returns and the useful factor are drawn from a multivariate normal distribution with a covariance matrix set equal to the estimated covariance matrix from the 1959:2–2012:12 sample of monthly gross returns on the 25 Fama-French portfolios and the value-weighted market excess return
- For misspecified models, the means of the simulated returns are set equal to the means of the actual returns
- For correctly specified models, the means of the returns are set such that the asset-pricing model restrictions are satisfied (i.e., the pricing errors are zero)
- The mean of the simulated useful factor is calibrated to the mean of the market excess return

Robotti (2015)
Misspecification and Weak Identification
May 18, 2015
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  - the mean of the simulated useful factor is calibrated to the mean of the market excess return
- The spurious factor is generated as a standard normal random variable which is uncorrelated with the returns and the useful factor
Simulation evidence: CU-GMM

Rejection rates of specification test and $t$-tests of statistical significance

<table>
<thead>
<tr>
<th></th>
<th>$t_1$ (useful)</th>
<th>$t_2$ (spurious)</th>
<th>$\mathcal{J}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>5%</td>
<td>1%</td>
<td>10% 5% 1%</td>
</tr>
<tr>
<td>(1)</td>
<td>0.953</td>
<td>0.936</td>
<td>1.00 1.00 0.999</td>
</tr>
</tbody>
</table>

Notes: (1) denotes the model with a useful factor only. The model is misspecified. The sample size is $T = 600$ and the number of Monte Carlo replications is 100,000.
Simulation evidence: CU-GMM

Rejection rates of specification test and $t$-tests of statistical significance

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<thead>
<tr>
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<th>$t_1$ (useful)</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10% 5% 1%</td>
<td>10% 5% 1%</td>
<td>10% 5% 1%</td>
</tr>
<tr>
<td>(1)</td>
<td>0.953 0.936 0.889</td>
<td>- - -</td>
<td>1.00 1.00 0.999</td>
</tr>
<tr>
<td>(2)</td>
<td>- - -</td>
<td>1.00 1.00 1.00</td>
<td>0.105 0.052 0.011</td>
</tr>
</tbody>
</table>

**Notes:** (1) denotes the model with a useful factor only and (2) denotes the model with a spurious factor only. The model is misspecified. The sample size is $T = 600$ and the number of Monte Carlo replications is 100,000.
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Rejection rates of specification test and $t$-tests of statistical significance

<table>
<thead>
<tr>
<th></th>
<th>$t_1$ (useful)</th>
<th>$t_2$ (spurious)</th>
<th>$\mathcal{J}$ Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.953 0.936 0.889</td>
<td>- - -</td>
<td>1.00 1.00 0.999</td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td>1.00 1.00 1.00</td>
<td>0.105 0.052 0.011</td>
</tr>
<tr>
<td>1%</td>
<td>0.171 0.096 0.024</td>
<td>1.00 1.00 1.00</td>
<td>0.085 0.040 0.007</td>
</tr>
</tbody>
</table>

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Simulation evidence: specification tests (useful factor)

Rejection Rates of Specification Tests

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\mathcal{J}$ Test (CU-GMM)</th>
<th>$\mathcal{S}$ Test (ML)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10% 5% 1%</td>
<td>10% 5% 1%</td>
</tr>
<tr>
<td>200</td>
<td>0.214 0.131 0.040</td>
<td>0.149 0.081 0.019</td>
</tr>
<tr>
<td>600</td>
<td>0.135 0.072 0.017</td>
<td>0.113 0.059 0.013</td>
</tr>
<tr>
<td>3600</td>
<td>0.105 0.054 0.011</td>
<td>0.103 0.052 0.011</td>
</tr>
</tbody>
</table>

Correctly Specified Model (Useful Factor)

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\mathcal{J}$ Test (CU-GMM)</th>
<th>$\mathcal{S}$ Test (ML)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10% 5% 1%</td>
<td>10% 5% 1%</td>
</tr>
<tr>
<td>200</td>
<td>0.900 0.831 0.635</td>
<td>0.866 0.781 0.557</td>
</tr>
<tr>
<td>600</td>
<td>1.000 1.000 0.999</td>
<td>1.000 1.000 0.998</td>
</tr>
<tr>
<td>3600</td>
<td>1.000 1.000 1.000</td>
<td>1.000 1.000 1.000</td>
</tr>
</tbody>
</table>

Misspecified Model (Useful Factor)
Simulation evidence: specification tests (spurious factor)

Rejection Rates of Specification Tests

<table>
<thead>
<tr>
<th></th>
<th>$J$ Test (CU-GMM)</th>
<th>$S$ Test (ML)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>10%</td>
<td>5%</td>
</tr>
<tr>
<td>200</td>
<td>0.030</td>
<td>0.011</td>
</tr>
<tr>
<td>600</td>
<td>0.010</td>
<td>0.003</td>
</tr>
<tr>
<td>3600</td>
<td>0.006</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Correctly Specified Model (Spurious Factor)

Misspecified Model (Spurious Factor)
Simulation evidence: HJ-distance test (spurious factor)

Rejection Rates of HJ-Distance Test (non-optimal GMM)

<table>
<thead>
<tr>
<th></th>
<th>$N = 11$</th>
<th></th>
<th>$N = 26$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
<td>5%</td>
<td>1%</td>
</tr>
<tr>
<td>$T$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.091</td>
<td>0.046</td>
<td>0.010</td>
</tr>
<tr>
<td>600</td>
<td>0.076</td>
<td>0.036</td>
<td>0.007</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.070</td>
<td>0.033</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Correctly Specified Model (Spurious Factor)

|       |          |       |          |          |       |          |
| 200   | 0.334    | 0.228 | 0.092    | 0.970    | 0.949 | 0.878    |
| 600   | 0.692    | 0.597 | 0.395    | 0.996    | 0.994 | 0.987    |
| $\infty$ | 0.944 | 0.931 | 0.902    | 0.999    | 0.999 | 0.998    |

Misspecified Model (Spurious Factor)
Simulation evidence: $R^2$ (CU-GMM)

Empirical Distribution of the $R^2$ coefficient (CU-GMM)

<table>
<thead>
<tr>
<th>$T$</th>
<th>mean</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>50%</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.298</td>
<td>0.000</td>
<td>0.003</td>
<td>0.012</td>
<td>0.251</td>
<td>0.669</td>
<td>0.755</td>
<td>0.871</td>
</tr>
<tr>
<td>600</td>
<td>0.214</td>
<td>0.000</td>
<td>0.003</td>
<td>0.011</td>
<td>0.176</td>
<td>0.481</td>
<td>0.563</td>
<td>0.692</td>
</tr>
<tr>
<td>3600</td>
<td>0.172</td>
<td>0.012</td>
<td>0.041</td>
<td>0.062</td>
<td>0.164</td>
<td>0.293</td>
<td>0.332</td>
<td>0.404</td>
</tr>
</tbody>
</table>

Misspecified Model (Useful Factor)

Misspecified Model (Spurious Factor)

<table>
<thead>
<tr>
<th>$T$</th>
<th>mean</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>50%</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.900</td>
<td>0.342</td>
<td>0.658</td>
<td>0.770</td>
<td>0.944</td>
<td>0.983</td>
<td>0.988</td>
<td>0.993</td>
</tr>
<tr>
<td>600</td>
<td>0.989</td>
<td>0.929</td>
<td>0.966</td>
<td>0.976</td>
<td>0.993</td>
<td>0.998</td>
<td>0.998</td>
<td>0.999</td>
</tr>
<tr>
<td>3600</td>
<td>1.000</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Simulation evidence: \( t \)-tests (useful factor)

<table>
<thead>
<tr>
<th>( T )</th>
<th>( \text{Correctly Specified (Useful Factor)} )</th>
<th>( \text{Misspecified (Useful Factor)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.319 0.238 0.123 0.449 0.362 0.217</td>
<td>0.632 0.565 0.442 0.849 0.814 0.732</td>
</tr>
<tr>
<td>600</td>
<td>0.153 0.089 0.025 0.533 0.423 0.230</td>
<td>0.459 0.377 0.245 0.953 0.936 0.889</td>
</tr>
<tr>
<td>3600</td>
<td>0.109 0.056 0.012 0.987 0.973 0.904</td>
<td>0.368 0.284 0.159 1.000 1.000 1.000</td>
</tr>
</tbody>
</table>

Note: \( \lambda^* \) is the (pseudo-) true value of the parameter.

Rejection Rates of \( t \)-tests (CU-GMM)

\[
H_0 : \lambda = \lambda^* \\
H_0 : \lambda = 0
\]

\[
\begin{array}{ccc|ccc}
T & 10\% & 5\% & 1\% & 10\% & 5\% & 1\% \\
\hline
200 & 0.319 & 0.238 & 0.123 & 0.449 & 0.362 & 0.217 \\
600 & 0.153 & 0.089 & 0.025 & 0.533 & 0.423 & 0.230 \\
3600 & 0.109 & 0.056 & 0.012 & 0.987 & 0.973 & 0.904 \\
\end{array}
\]
**Simulation evidence: \( t \)-tests (spurious factor)**

Rejection Rates of \( t \)-tests (CU-GMM)

\[ H_0 : \lambda = 0 \]

<table>
<thead>
<tr>
<th>( T )</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correctly Specified (Spurious Factor)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.850</td>
<td>0.818</td>
<td>0.749</td>
</tr>
<tr>
<td>600</td>
<td>0.813</td>
<td>0.774</td>
<td>0.691</td>
</tr>
<tr>
<td>3600</td>
<td>0.800</td>
<td>0.758</td>
<td>0.668</td>
</tr>
<tr>
<td>Misspecified (Spurious Factor)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.997</td>
<td>0.996</td>
<td>0.994</td>
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</tr>
</tbody>
</table>
Similar results continue to hold for the case of a weak factor (a factor that is weakly correlated with returns).

Similar results also arise when none of the factors is spurious but two or more factors are noisy (mismeasured) versions of the same underlying (latent) factor.

- An example of this scenario is a consumption-based asset-pricing model whose empirical specification includes several measures of consumption growth (possibly based on non-durables, durables, garbage, electricity consumption, etc.).
- In this case, the full rank condition is violated and the limiting representations for the noisy factors are the same as the asymptotic distribution for the spurious factor.
What has been done


- **Globally misspecified and unidentified models**: Kan and Zhang (1999a, 1999b); Kleibergen (2009), Kleibergen and Zhan (2014), Gospodinov, Kan, and Robotti (2014, 2015b)
Concluding remarks

- The results from many popular empirical asset-pricing models may be spurious
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- The spurious results in these models (almost perfect fit and strong evidence of nonzero risk premia) arise from the combined effect of identification failure and model misspecification.
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It is important to stress that this is not an isolated problem limited to a particular sample (data frequency), test assets, and asset-pricing models, which suggests that the statistical evidence on the pricing ability of many macro factors and their usefulness in explaining the cross-section of asset returns should be interpreted with caution.
Concluding remarks

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- Some warning signs about this problem (for example, the outcome of a rank test) are often ignored by applied researchers.
Concluding remarks

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- The spurious results in these models (almost perfect fit and strong evidence of nonzero risk premia) arise from the combined effect of identification failure and model misspecification.
- It is important to stress that this is not an isolated problem limited to a particular sample (data frequency), test assets, and asset-pricing models, which suggests that the statistical evidence on the pricing ability of many macro factors and their usefulness in explaining the cross-section of asset returns should be interpreted with caution.
- Some warning signs about this problem (for example, the outcome of a rank test) are often ignored by applied researchers.
- While non-invariant estimators (HJ-distance non-optimal GMM, OLS/GLS two-pass regression) also suffer from similar problems, the invariant estimators (CU-GMM, ML) turn out to be much more sensitive to model misspecification and lack of identification.