Misspecification and Weak Identification in Asset-Pricing Models

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Outline

1. Preliminaries
2. Setup
3. Severity of the problem
4. What has been done
5. Conclusion

(Global or Fixed) Model Misspecification

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3. As a result, standard error (confidence interval) construction and model comparison should allow for possible model misspecification. Still, it is common practice to use GMM standard errors for correctly specified models even when the model is rejected by the data

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1. Asset returns are very noisy and only weakly correlated with the much less volatile macroeconomic (non-traded) factors. The average (absolute) correlations of 25 FF portfolio monthly returns with *vw* (0.82), c_{nd} (0.14), c_d (0.05), *lab* (0.04), $c \cdot cay$ (0.01)

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   - The presence of a spurious factor does affect the inference on the remaining parameters and the test of correct model specification

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$$\bar{e}(\lambda)=\frac{1}{T}\sum_{t=1}^{T} e_t(\lambda) \text{ and } \hat{W}(\lambda)=\frac{1}{T}\sum_{t=1}^{T}(e_t(\lambda)\text{-}\bar{e}(\lambda))(e_t(\lambda)\text{-}\bar{e}(\lambda))'$$

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- The over-identifying restriction test of the asset-pricing model is

$$\mathcal{J}(\lambda) = T\min_\lambda \bar{e}(\lambda)'\hat{W}(\lambda)^{-1}\bar{e}(\lambda)$$

and $\mathcal{J}(\hat{\lambda}) \to^d \chi^2_{N-K}$ when the asset-pricing model holds

Beta-pricing model: ML framework

- It is often desirable to estimate and evaluate the asset-pricing model in the beta-pricing setup

$$R_t - \mu = \beta(f_t - \mu_f) + \epsilon_t, \quad t = 1, \ldots, T,$$

where $\mu = E[R_t]$, $\mu_f = E[f_t]$ and β is an $N \times (K-1)$ matrix

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- Assuming that $\epsilon_t \sim iid\mathcal{N}(0_N, \Sigma)$ conditional on f_t, the MLE of γ is

$$\hat{\gamma} = \text{argmin}_{\gamma} \frac{(\hat{\mu} - 1_N\gamma_0 - \hat{\beta}\gamma_1)'\hat{\Sigma}^{-1}(\hat{\mu} - 1_N\gamma_0 - \hat{\beta}\gamma_1)}{1 + \gamma_1' \hat{V}_f^{-1}\gamma_1},$$

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- The test of correct model specification is given by

$$\mathcal{S} = TQ(\hat{\gamma}) \xrightarrow{d} \chi^2_{N-K}$$

  and the fitted returns are constructed as $\hat{\mu} = 1_N\hat{\gamma}_0 + \hat{\beta}\hat{\gamma}_1$

Main statistics of interest

1. t-statistics of statistical significance which provide evidence on whether
   1. the factor is priced (SDF framework, $H_0 : \lambda_i = 0$)
   2. the risk premium is nonzero (beta-pricing model, $H_0 : \gamma_i = 0$)
2. Test for correct model specification
   1. $\mathcal{J}$ test for overidentifying restrictions (SDF: $H_0 : D\lambda - 1_N = 0_N$)
   2. $\mathcal{S}$ (Shanken's) test (beta-pricing model, $H_0 : \mu - 1_N\gamma_0 - \beta\gamma_1 = 0_N$)
3. Goodness-of-fit (pseudo-R^2) measure: squared correlation between actual and fitted ($\hat{\mu} = 1_N\hat{\gamma}_0 + \hat{\beta}\hat{\gamma}_1$) returns
   - the fitted returns in the SDF approach are constructed using the mapping between the SDF and beta-pricing model parameters

$$\gamma_0 = \frac{1}{\lambda_0 + \mu_f'\lambda_1}, \gamma_1 = \frac{-V_f\lambda_1}{\lambda_0 + \mu_f'\lambda_1}$$

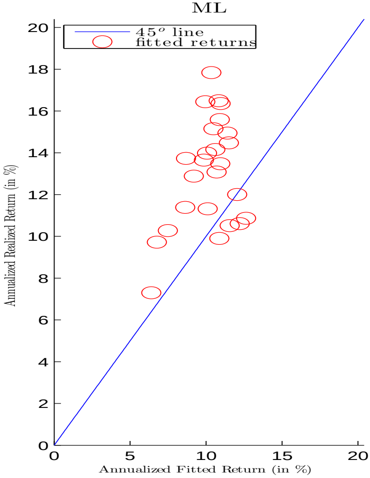
Empirical evidence: CAPM

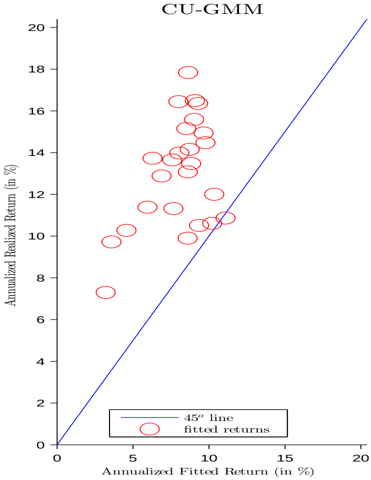
Empirical results from estimating CAPM (market factor)
(test assets: 25 Fama-French portfolio returns, 1959:02 – 2012:12)

	|t-stat|	spec. test	R^2
ML	3.24 (0.0012)	67.66 (0.0000)	0.1346
CU-GMM	4.29 (0.0000)	64.58 (0.0000)	0.1999

Notes: The specification test is the $\mathcal{S}$ test (ML) or $\mathcal{J}$ test (CU-GMM) of the null of correct model specification. The R^2 is computed as the squared correlation of the actual and fitted returns. p-values in parentheses

Realized vs. fitted returns: CAPM





Empirical evidence: CAPM + "mysterious" risk factor

Empirical results from estimating CAPM + "mysterious" risk factor (test assets: 25 Fama-French portfolio returns, 1959:02 – 2012:12)

	market factor	"mysterious" factor		
	|t-stat|	|t-stat|	spec. test	R^2
ML	1.03 (0.3030)	4.61 (0.0000)	19.94 (0.5871)	0.9972
CU-GMM	1.36 (0.1738)	4.42 (0.0000)	18.07 (0.7019)	0.9542

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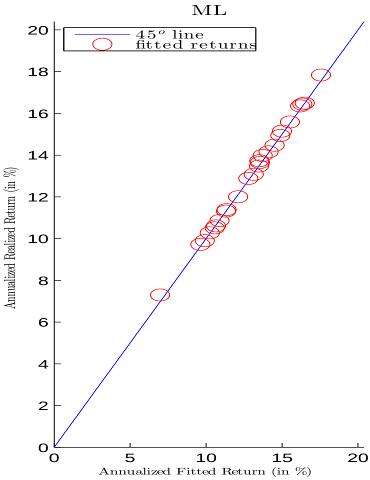

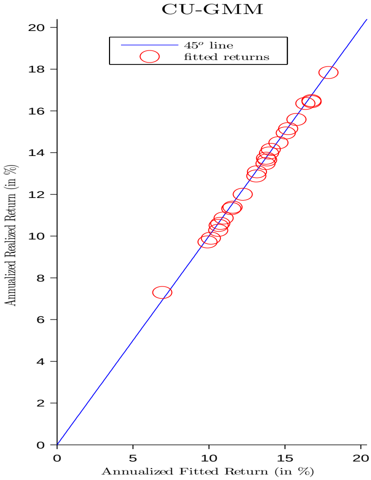

Empirical evidence: model with "mysterious" risk factor

Empirical results from estimating a model with a "mysterious" risk factor (test assets: 25 Fama-French portfolio returns, 1959:02 – 2012:12)

	|t-stat|	spec. test	R^2
ML	4.67 (0.0000)	21.36 (0.5592)	0.9998
CU-GMM	4.51 (0.0000)	19.95 (0.6447)	0.9941

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- In summary, an arbitrarily bad model with one or more factors that are independent of asset returns (*spurious* factors) is concluded to be a correctly specified model with a spectacular fit and priced risk factors
- Even worse, the priced factors that are highly correlated with returns are driven out (become insignificant) when a spurious factor is included in the model

Further empirical evidence: ML

- These results also arise in popular empirical models
  - C-LAB: market factor, growth rate of per capita labor income (*labor*), and the lagged default premium (*prem*)
  - CC-CAY: real per capita consumption growth (*cg*), the lagged consumption-aggregate wealth ratio (*cay*), and an interaction term between these two factors ($cg \cdot cay$)

	C-LAB	CC-CAY
$\mathcal{S}$ (*p*-value)	23.10 (0.3388)	11.58 (0.9503)
t_{market}	1.34	
t_{labor}	2.81	
t_{prem}	4.21	
t_{cg}		0.90
t_{cay}		0.76
$t_{cg \cdot cay}$		3.45
R^2	0.9994	0.9997

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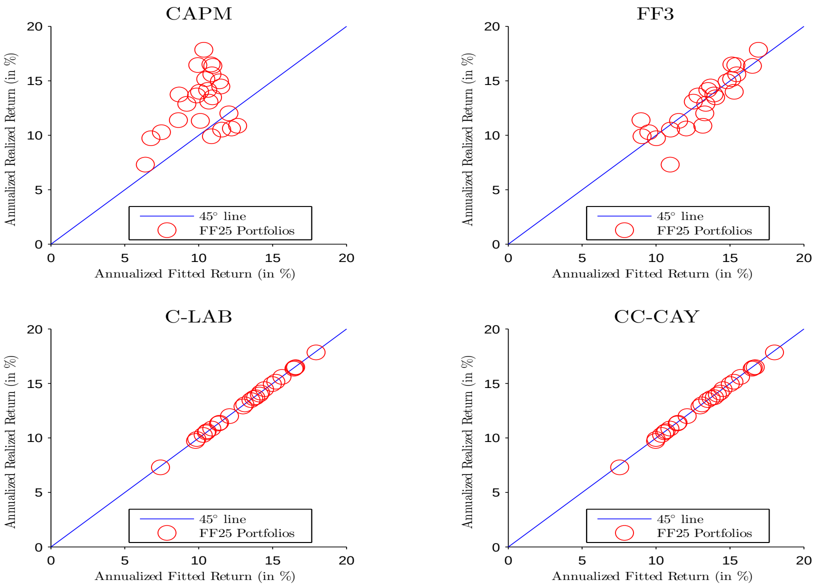

Source of the problem

- In all of these examples (model with "mysterious" factor, C-LAB, CC-CAY), there is identification failure
- In the SDF framework, the matrix

$$\frac{\partial \bar{e}(\lambda)}{\partial \lambda'} = E[R_t \tilde{f}_t'] = D$$

  is of reduced rank because one or more factors are uncorrelated (or weakly correlated in finite samples) with returns
- The rank test (Cragg-Donald) cannot reject the null that the second moment matrix of the returns and the factors is of reduced rank
- For example,

	CAPM	FF3	C-LAB	CC-CAY
Rank test (p-value)	133.43 (0.0000)	86.18 (0.0000)	20.82 (0.5320)	10.44 (0.9818)

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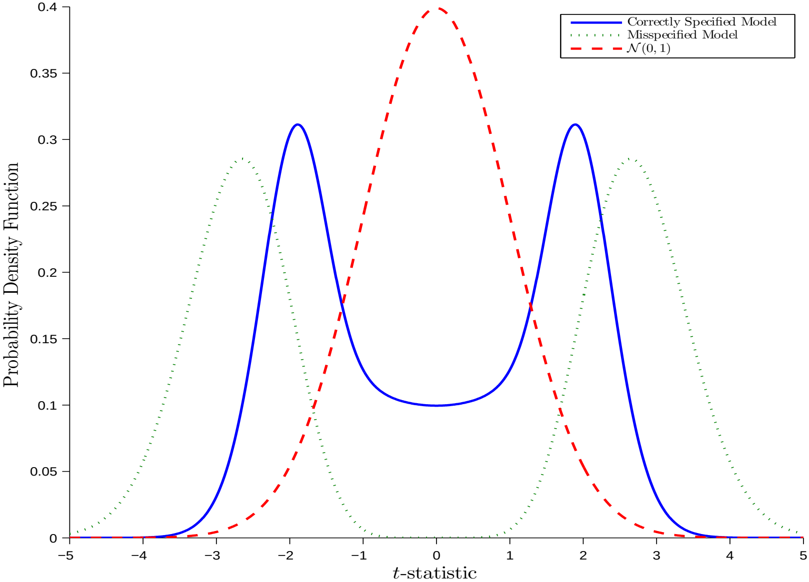
4. Risk factors, that are useful and priced, are likely to be deemed unpriced
   - the t-tests of statistical significance for the useful factors are inconsistent (i.e., converge to bounded random variables) and exhibit power that is close to the size of the test

Limiting densities of the t-statistics: CU-GMM



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  - the mean of the simulated useful factor is calibrated to the mean of the market excess return
- The spurious factor is generated as a standard normal random variable which is uncorrelated with the returns and the useful factor

Simulation evidence: CU-GMM

Rejection rates of specification test and t-tests of statistical significance

	t_1 (useful)			t_2 (spurious)			$\mathcal{J}$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
(1)	0.953	0.936	0.889	-	-	-	1.00	1.00	0.999

Notes: (1) denotes the model with a useful factor only. The model is misspecified. The sample size is $T = 600$ and the number of Monte Carlo replications is 100,000

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Rejection rates of specification test and t-tests of statistical significance

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(2)	-	-	-	1.00	1.00	1.00	0.105	0.052	0.011
(3)	0.171	0.096	0.024	1.00	1.00	1.00	0.085	0.040	0.007

Notes: (1) denotes the model with a useful factor only, (2) denotes the model with a spurious factor only, and (3) denotes the model with one useful factor and one spurious factor. The model is misspecified. The sample size is $T = 600$ and the number of Monte Carlo replications is 100,000

Simulation evidence: specification tests (useful factor)

Rejection Rates of Specification Tests

T	$\mathcal{J}$ Test (CU-GMM) 10%	5%	1%	$\mathcal{S}$ Test (ML) 10%	5%	1%
Correctly Specified Model (Useful Factor)						
200	0.214	0.131	0.040	0.149	0.081	0.019
600	0.135	0.072	0.017	0.113	0.059	0.013
3600	0.105	0.054	0.011	0.103	0.052	0.011
Misspecified Model (Useful Factor)						
200	0.900	0.831	0.635	0.866	0.781	0.557
600	1.000	1.000	0.999	1.000	1.000	0.998
3600	1.000	1.000	1.000	1.000	1.000	1.000

Simulation evidence: specification tests (spurious factor)

Rejection Rates of Specification Tests

	$\mathcal{J}$ Test (CU-GMM)			$\mathcal{S}$ Test (ML)		
T	10%	5%	1%	10%	5%	1%
Correctly Specified Model (Spurious Factor)						
200	0.030	0.011	0.001	0.014	0.004	0.000
600	0.010	0.003	0.000	0.007	0.002	0.000
3600	0.006	0.001	0.000	0.005	0.001	0.000
Misspecified Model (Spurious Factor)						
200	0.130	0.063	0.011	0.105	0.050	0.008
600	0.113	0.057	0.011	0.105	0.052	0.011
3600	0.103	0.052	0.010	0.103	0.052	0.010

Simulation evidence: HJ-distance test (spurious factor)

Rejection Rates of HJ-Distance Test (non-optimal GMM)

	$N = 11$			$N = 26$		
T	10%	5%	1%	10%	5%	1%
	Correctly Specified Model (Spurious Factor)					
200	0.091	0.046	0.010	0.190	0.115	0.036
600	0.076	0.036	0.007	0.110	0.058	0.013
∞	0.070	0.033	0.006	0.079	0.038	0.007
	Misspecified Model (Spurious Factor)					
200	0.334	0.228	0.092	0.970	0.949	0.878
600	0.692	0.597	0.395	0.996	0.994	0.987
∞	0.944	0.931	0.902	0.999	0.999	0.998

Simulation evidence: R^2 (CU-GMM)

Empirical Distribution of the R^2 coefficient (CU-GMM)

T	mean	1%	5%	10%	50%	90%	95%	99%
	Misspecified Model (Useful Factor)							
200	0.298	0.000	0.003	0.012	0.251	0.669	0.755	0.871
600	0.214	0.000	0.003	0.011	0.176	0.481	0.563	0.692
3600	0.172	0.012	0.041	0.062	0.164	0.293	0.332	0.404
	Misspecified Model (Spurious Factor)							
200	0.900	0.342	0.658	0.770	0.944	0.983	0.988	0.993
600	0.989	0.929	0.966	0.976	0.993	0.998	0.998	0.999
3600	1.000	0.999	0.999	0.999	1.000	1.000	1.000	1.000

Simulation evidence: t-tests (useful factor)

Rejection Rates of t-tests (CU-GMM)

T	$H_0: \lambda = \lambda^*$ 10%	5%	1%	$H_0: \lambda = 0$ 10%	5%	1%
	Correctly Specified (Useful Factor)					
200	0.319	0.238	0.123	0.449	0.362	0.217
600	0.153	0.089	0.025	0.533	0.423	0.230
3600	0.109	0.056	0.012	0.987	0.973	0.904
	Misspecified (Useful Factor)					
200	0.632	0.565	0.442	0.849	0.814	0.732
600	0.459	0.377	0.245	0.953	0.936	0.889
3600	0.368	0.284	0.159	1.000	1.000	1.000

Note: λ^* is the (pseudo-) true value of the parameter

Simulation evidence: t-tests (spurious factor)

Rejection Rates of t-tests (CU-GMM)
$H_0 : \lambda = 0$

T	10%	5%	1%
Correctly Specified (Spurious Factor)			
200	0.850	0.818	0.749
600	0.813	0.774	0.691
3600	0.800	0.758	0.668
Misspecified (Spurious Factor)			
200	0.997	0.996	0.994
600	1.000	1.000	1.000
3600	1.000	1.000	1.000

Remarks

- Similar results continue to hold for the case of a weak factor (a factor that is weakly correlated with returns)
- Similar results also arise when none of the factors is spurious but two or more factors are noisy (mismeasured) versions of the same underlying (latent) factor
    - An example of this scenario is a consumption-based asset-pricing model whose empirical specification includes several measures of consumption growth (possibly based on non-durables, durables, garbage, electricity consumption, etc.)
    - In this case, the full rank condition is violated and the limiting representations for the noisy factors are the same as the asymptotic distribution for the spurious factor

What has been done

- **Globally misspecified and identified models**: White (1982, 1984), Hall and Inoue (2003), Hansen, Heaton, and Luttmer (1995), Hou and Kimmel (2006), Shanken and Zhou (2007), Kan and Robotti (2008, 2009), Kan, Robotti, and Shanken (2013), Gospodinov, Kan, and Robotti (2013, 2015a)
- **Correctly specified and unidentified models**: Kan and Zhang (1999a, 1999b); Stock and Wright (2000), Andrews and Chen (2012), Kleibergen (2009), Kleibergen and Zhan (2014), Gospodinov, Kan, and Robotti (2014, 2015b), Burnside (2011, 2014) Khalaf and Schaller (2014), Bryzgalova (2014), Manresa, Peñaranda, and Sentana (2014)
- **Globally misspecified and unidentified models**: Kan and Zhang (1999a, 1999b); Kleibergen (2009), Kleibergen and Zhan (2014), Gospodinov, Kan, and Robotti (2014, 2015b)

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- Some warning signs about this problem (for example, the outcome of a rank test) are often ignored by applied researchers
- While non-invariant estimators (HJ-distance non-optimal GMM, OLS/GLS two-pass regression) also suffer from similar problems, the invariant estimators (CU-GMM, ML) turn out to be much more sensitive to model misspecification and lack of identification